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Macroeconomics | Spring 2014

Homework 3

Instructions: I encourage you to work in groups of 3 to 4 people. But please hand in your own work. This homework is due in Diego's hands on Thursday, February 20.

Part I.

Please solve exercises 14.5, 14.9, 14.10, and 11.5.

Part II.

In answering this question, please feel free study chapter 13 and/or the Stachurski-Sargent quant-econ lecture on asset pricing within the Lucas model in a finite-state Markov setting.

Setup:

Consider a pure-exchange, representative agent economy. A representative consumer ranks stochastic processes for consumption of a single non-durable good $\{C_t\}_{t=0}^{\infty}$ according to

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t),$$

where $\beta \in (0,1)$ and $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$. The representative consumer's endowment obeys

$$C_{t+1} = \lambda_{t+1} C_t$$

where λ_{t+1} is described by a finite state Markov chain with transition matrix

$$P_{ij} = \text{Prob}(\lambda_{t+1} = \bar{\lambda}_j | \lambda_t = \bar{\lambda}_i)$$

and $\forall t \geq 0$, $\lambda_t \in \{\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_n\}$.

a. Assume that there are complete markets in one-step-ahead Arrow securities. Compute equilibrium prices of one-step-ahead Arrow securities.

b. Compute what prices would be for two-step-ahead Arrow securities if these markets were open too.

c. Describe how to compute an equilbrium *pricing function* of a "Lucas tree," that is, an *ex dividend* claim to the endowment. In particular, using a "resolvent operator," give a formula for the pricing function cast entirely in terms of matrices.

d. An infinite horizon call option on the Lucas tree entitles the owner of the option to purchase the tree (ex this period's dividend) at any time from now on at a *strike price* $\check{p}$. Please write a Bellman equation for the function that prices an infinite horizon call option on the Lucas tree. Describe an iterative algorithm that you could use to compute this function, using matrix algebra only.

e. Extra credit: (Nostalgia from John Leahy's class) Please how the algorithm that you described in your answer to part **d** uses a contraction mapping. Verify that the mapping in question is a contraction.