Productivity Losses from the Attention to Aggregate Uncertainty

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Abstract

I present a static model of an economy where firms allocate attention in order to reduce the uncertainty on aggregate monetary shocks, with a cost in terms of productivity. There is a money-holding representative household with Dixit-Stiglitz (1977) preferences and endogenous labour. Money is neutral when there is no uncertainty about the shock, which can be due to either endogenous or exogenous conditions. As uncertainty increases, more attention is drawn from productivity towards the understanding of the aggregate conditions, leading to a reduction in aggregate output. The result provides a rationale for the relationship between monetary volatility and aggregate output in Latin-American countries. I calibrate the model using data from Ecuador and find the model is able to explain almost half of the output fluctuations observed. The model suggests that the cost of monetary uncertainty in terms of consumption can be as high as 13% in a single year, with an annual average cost of 8%.

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1 Introduction

Economists have always studied how decision-makers allocate scarce resources. However, only recently has attention allocation become of particular interest to economists. The recent literature on rational inattention[1] or bounded rationality[2], studies how decision-makers allocate the scarce resource attention. If agents have a limited amount of attention, they have to decide optimally on how to allocate it. Even if there is a vast amount of information available, due to this limit it is simply impossible to acquire all this information. Therefore, decision-makers have to choose which information to attend to carefully, which to attend to less carefully, and which to completely ignore. However, the implications of paying too much attention to aggregate uncertainty on productivity have not been studied. In this paper, I entertain the hypothesis that if entrepreneurs have to pay attention both to understand macro-aggregate conditions (*e.g.* inflation or exchange rate) - in order to do an optimal pricing for example - and to increase productivity (like Kirzner's "alertness"), the limited information-processing capacity should have important effects. Therefore, I study how the amount of volatility of macroeconomic conditions affects this trade off and its consequences on the levels of productivity and output.

Table 1: Expected Sales Growth and Uncertainty in WBES (2000)

VARIABLES	RE	RE	RE	FE	FE	FE
Economic Unpredictability	-2.108***		-1.889***	-2.190***		-2.039***
	(0.646)		(0.566)	(0.700)		(0.619)
Policy Unpredictability		-1.246***	-0.260		-1.649**	-0.192
		(0.426)	(0.666)		(0.726)	(0.680)
Observations	5,404	5,548	5,352	5,404	5,548	5,352
R-squared	0.007	0.004	0.007	0.007	0.005	0.007
Number of countries	53	69	53	53	69	53
Company characteristics	Y	Y	Y	Y	Y	Y
Country characteristics	Y	Y	Y	N	N	N
Legal Origin	Y	Y	Y	N	N	N

*** p<0.01, ** p<0.05, * p<0.1.

Robust standard errors in parentheses. Company characteristics: Foreign Owned, Government owned. Country characteristics: Initial GDP , GDP growth.

[1] See Sims (2003) or Mackowiak and Wiederholt (2011).

[2] See Gabaix (2011).

If we had perfect data, we would like to test the relationship between uncertainty and attention allocation. Unfortunately, this kind of information is not available.[3] Hence, a second alternative is to test some of the consequences of the hypothesis presented above. Table 1 shows the relationship between expected sales growth and country-level uncertainty using data from the World Business Environment Survey in 2000. In this survey several businessmen from different countries were interviewed and asked about their own business conditions and their countries' ones. Between these, two questions are of particular interest since they evaluate the economic and policy unpredictability of their countries. Table 1 shows that the relation between this uncertainty and expected sales growth is clearly negative and stable along the several regression specifications (using only one uncertainty proxy or both, and under country random effects or country fixed effects). Since surveys can have weak results due to the noise present in them, Table 2 shows the results from doing a within groups regression, using 5-year-average country data between 1960 and 2010, of real GDP per capita growth on three proxies for economic uncertainty: the standard deviation of inflation, the exchange rate and money (M2) growth (using the log of them in order to allow for a possible non-linear relationship). It is seen that the relation is once again significantly negative and stable along the different specifications.

These results suggest that growth is significantly negatively correlated with uncertainty. Although subject to several possible econometric problems (*e.g.* endogeneity), they do provide sufficient evidence in order to start a research project on this subject. Moreover, due to these possible problems, doing a structural approach - like the one introduced in this article - to the question of interest seems more useful. It is important to clarify that many models (some of which are discussed below) could potentially explain this qualitative relationship between uncertainty and output. For example, the real option literature suggests that increased uncertainty discourages actions such as investment and hiring. The goal here is not to diminish the importance of other channels, but rather to explore the viability of an alternative (or complementary) explanation for the observed negative correlation between growth and (particularly monetary) uncertainty. An interesting extension would be to build a model that allows for different mechanisms to compete and then quantitatively test their relative importance.

[3] It would be interesting to see information on business entrepreneurs time allocation across periods with different levels of uncertainty. However, I have not been able to find this kind of data.

Table 2: GDP Growth and Uncertainty, Within Groups Regression

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
SD Inflation	-1.100***			-0.560***	-0.649***	-0.503***
	(0.149)			(0.145)	(0.147)	(0.189)
SD Exchange Rate		-0.259***		-0.210*	-0.182*	-0.231**
		(0.097)		(0.109)	(0.100)	(0.100)
SD M2 Growth			-0.628***	-0.171	-0.134	-0.258**
			(0.127)	(0.122)	(0.121)	(0.115)
Observations	937	1,058	892	752	740	657
R-squared	0.370	0.215	0.224	0.293	0.315	0.419
Number of countries	135	137	129	119	117	108
Population	N	N	N	Y	Y	Y
Government	N	N	N	N	Y	Y
Economics	N	N	N	N	N	Y

*** p<0.01, ** p<0.05, * p<0.1

Robust standard errors in parentheses. All regressions control for year effects. Population: Population, Population growth and Education. Government: Government expenditure. Economics: Trade, Investment, Inflation, Trade.

I build a static model of an economy where firms have to allocate attention in order to reduce the uncertainty on an aggregate monetary shock, with a cost in terms of productivity, which provides a rationale for the relationship observed between monetary volatility and aggregate output in Latin-American countries. Similarly to the intuition in the Rational Inattention literature, as uncertainty increases, more attention is drawn towards its reduction. However, in this model, this has a cost in terms of productivity, leading to a reduction in aggregate output. Applying the model to Latin-American countries seems reasonable since monetary uncertainty is a major source of policy uncertainty there. Therefore, I calibrate the model using data from Ecuador and find the model is able to explain almost half of the output fluctuations observed. Probably more importantly for policy analysis, the model suggests that the levels of monetary uncertainty observed in the data can be very costly in terms of consumption, generating losses of up to 13% in only one year and an annual average cost of 8%. I also apply the calibrated parameters to evaluate the cost of uncertainty in Argentina, Chile and Mexico and find consistent results. It should be remarked that this cost is significantly higher than what has been found in the previous literature on the effects of stabilization on the level of consumption, as described in the next section.

The rest of the paper is organized as follows. Section 2 presents the previous literature findings on the relationship between uncertainty and welfare. Section 3 introduces the model. Section 4 characterizes the model's equilibrium and implications for a given set of parameters. Section 5 provides the calibration strategy and the quantitative analysis of the effect of monetary instability

on aggregate output for four Latin-American countries. Section 6 discusses possible extensions and concludes. The appendices contain the proofs of the analytical results needed for the numerical solution, and the list of variables - and their sources - used in the empirical work.

2 Literature

Robert Lucas argued in his influential 1987 monograph *Models of Business Cycles* that deviations from stable growth after the WWII in the United States were actually a minor concern. However, Lucas asserted that stabilization would only eliminate deviations from trend, implying consumption would revert to its average level. On the other hand, various economist have since noted that the level or the growth rate of consumption might change in response to stabilization.

Among those who believe stabilization can change the level of consumption, DeLong and Summers (1988) argue that, differently from Lucas, stabilization would prevent economic activity from falling below its maximum potential. Following this view of stabilization, Ramey and Ramey (1991) argue that firms need to pre-commit to a specific technology before starting production. They build a model where these may include both long-run commitments, such as the determination of the scale of a new factory, and shorter-run commitments, such as the size of the attached labor force. Each technology corresponds to a different minimum efficient scale. In an uncertain environment, firms may end up with inappropriate technology for the scale of production they would have to undertake. Thus volatile environments are more likely to involve inefficient production. Differently, Mankiw (1988) and Yellen and Akerlof (2004) argue that shocks could affect the economy asymmetrically. Citing evidence that unemployment responds asymmetrically to changes in inflation, they argue that positive shocks boost economic activity less than negative shocks dampen it. Nevertheless, none of these papers suggests that stabilization would increase consumption by more than 2 percent.

Another possibility is that stabilization may affect the growth rate of consumption. The literature on "Managerial Myopia"[4] provides a model to explain evidence that short-term (*e.g.* takeover) pressures lead managers to focus more heavily on short-term profits rather than on long-term objectives. Auletta (1986) remarks: "In such a climate companies often find their attention diverted to short term, defensive stances...peddling assets [and] reducing long term capital investments in order to stretch fourth quarter earnings." The typical argument is based on uncertainty's effect on investment but this is misleading since the supply of assets is partially based on precautionary savings, an incentive to invest that would be eliminated without uncertainty. Even disregarding this issue,

[4]See, for example, Stein (1988, 1989)

increasing investment can reduce consumption significantly in the beginning, making stabilization's effect ambiguous. Matheron and Maury (2000) and Epaulard and Pommeret (2003) calculate the welfare cost of business cycles due to effects on investment to be of less than 0.5 percent. Finally, Barlevy (2004) argues that changes in investment affect growth asymmetrically: an increase in investment increases growth less than a similar decrease in investment decreases growth. Intuitively, this could reflect the inability of firms to undertake too many projects at once. Under such an environment, the effects are much larger: even without changing initial consumption, the cost of cycles is 7.5-8 percent of lifetime consumption. Thus, it is seen that previously suggested explanations in the literature on the effect of uncertainty on consumption is, except for Barlevy (2004), generally unable to generate the negative relation between uncertainty and output observed in the data.

In this paper I entertain the hypothesis that uncertainty has important effects on the economy through the role of the entrepreneur or firm manager. Fiet (1996) explains that in all four roles played by entrepreneurs that have been identified by scholars there is one element in common. They all include the acquisition of specific, risk-reducing information. Hayek (1945) remarks that the most valuable information to managers is that on the circumstances of the time and place of a particular deal. Moreover, Hayek also makes an extraordinary comment ahead of its time (which we can relate to the modern literature on rational inattention or bounded rationality):

> *To assume all the knowledge to be given to a single mind in the same manner in which we assume it to be given to us as the explaining economists is to assume the problem away and to disregard everything that is important and significant in the real world. [...] there is something fundamentally wrong with an approach which habitually disregards an essential part of the phenomena with which we have to deal: the unavoidable imperfection of man's knowledge and the consequent need for a process by which knowledge is constantly communicated and acquired.*

Volatility implies that information about the characteristics of the environment are constantly becoming obsolete, so that a continuous flow of new information is needed (Casson, 2005). Lippman and McCall (1979) remark that collecting information is costly and, particularly, very time-consuming. With the objective of explaining the significance of uncertainty on productivity, I combine the entrepreneurial perspective on the need for information with the literature on scarce resources (*e.g.* time) to incorporate information. I build a new alternative model which allows for the before mentioned novel interaction between paying attention to aggregate uncertainty and productivity. A model with a similar intuition can be seen in Takii (2009) where an entrepreneur has to decide how many signals to process (to increase the accuracy of her demand prediction). Takii also assumes that the higher the number of signals, the lower the productivity of the firm. However, the motivation and details of the model as well as its empirical application are different here. The next section will

describe my model more carefully.

3 Model

I present a static model with a representative money-holding consumer with Dixit-Stiglitz preferences and endogenous labour, and a continuum of goods produced monopolistically. Firms have to allocate attention in order to reduce the uncertainty on aggregate monetary shocks, but with a cost in terms of productivity. The model is based on two ideas. First, Lucas (1973) famous islands model or Cukierman (1984) extension, provide a useful starting point on the modeling framework of uncertainty and the effect of money supply shocks on equilibrium. Secondly, Mackowiak and Wiederholt (2011) explore the effect of rational inattention when aggregate and idiosyncratic shocks can take place in order to do a correct pricing. Although their modeling framework for uncertainty reduction through attention allocation is different from mine, the reasons to pay attention to aggregate shocks are similar. On the other hand, differently from them, the cost will be in terms of productivity since attention has to be withdrawn from it. This leads to its reduction as in any model of endogenous productivity where resources have to be spent for it to increase.[5] Therefore, enterpreneurs have to decide how much attention to pay to aggregate variables in order to reduce the uncertainty they have on them, taking into account the trade off in terms of productivity. The rest of this section is divided in three parts: Households, Monetary Policy and Firms.

Households

Households have to choose their consumption level (and, as in Dixit and Stiglitz (1977), how much of each variety to consume), their labour supply and their money holdings level, that maximize the utility function[6]

$$\max_{c_i,L,M} \left[\ln(C) + \gamma_m \ln\left(\frac{M}{P}\right) - \gamma_l \frac{L^{1+\Psi}}{1+\Psi} \right], \tag{1}$$

subject to the budget constraint

[5] See, for example, Romer (1986) or Aghion and Howitt (1992).

[6] An alternative (and equivalent) way to include money in the household problem would be with a cash in advance constraint (Feenstra, 1987)

$$M + PC = WL + D, \tag{2}$$

where consumption is aggregated as

$$C = \left[\int_0^1 c_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

and the usual aggregate price index can be derived to be

$$P = \left[\int_0^1 p_i^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \tag{4}$$

where $C, c_i, L, M, D, W, P, p_i$ correspond to the aggregate consumption, the consumption of good i, the labour supply, the money holdings, the firms' profits, the wage, the aggregate price and the price of good i, respectively. Notice that, for simplicity, there is no uncertainty at the Household level. Appendix A shows the solution to the Household's problem.

Monetary Policy

I assume the monetary policy follows a simple pattern as in Lucas (1973) or Cukierman (1984) since this is enough for the creation of the required monetary uncertainty[7]

$$M = \bar{M} e^{\varepsilon}, \tag{5}$$

where

$$\varepsilon \sim N\left(-\frac{\sigma_m^2}{2}, \sigma_m^2\right). \tag{6}$$

But why monetary uncertainty and not any other source of uncertainty? There are two main reasons for this choice. First, Lucas (1973) famous Island model uses money to show that nominal shocks can have real effects when people can't distinguish them perfectly. Even though his model could have used other forms of uncertainty, he used this one since it is a useful modeling tool. Its usefulness

[7]The distribution of the monetary shock is such that the process (5) is a mean-preserving spread.

relies on it being a policy variable that is clearly measurable as it is identifiable in the data and is not confused with other sources of uncertainty.[8] Secondly, Lucas (2003) finds that around 30 percent of variation in output can be attributed to monetary shocks in the US. I suspect that its importance should be relatively higher in Latin-American countries, which is the focus of the quantitative evaluation done in section 5, due to their higher variability of money growth.[9]

Firms

Each firm's production requires a combination of productivity and labour

$$y_i = A_i l_i^{\alpha}, \tag{7}$$

where y_i is the output, A_i is the productivity level and l_i is the labour input of firm i. In order to increase the productivity, firms have to pay attention to it

$$A_i = \bar{A}\left(1 + \eta Z_i\right), \tag{8}$$

where Z_i will be the time devoted to paying attention to productivity, and η its return measured in productivity levels.[10] Finally, since attention is a limited resource I normalize this resource such that

$$T_i + Z_i = 1, \tag{9}$$

showing that time is distributed between understanding macroeconomic conditions (T_i) or increasing productivity (Z_i). As suggested by Sims (2003), "the idea that individual people have limited capacity for processing information should not be controversial. It accords with ordinary experience,

[8] Notice that other alternative sources of uncertainty like labour regulation or output variance are, respectively, not identifiable and confused in the data.

[9] For example, while in the United States money grew at an annual rate of 7% with 2% standard deviation since 1960, in Argentina during the same period money grew at an annual rate of 60% with 42% standard deviation.

[10] An alternative (and possibly more consistent with the modeling of attention on monetary shocks) way would be for firms to receive a signal on their technology, which should be more accurate the more attention they pay to it. For example, the productivity could be

$$A_i = \bar{A}\left[1 - (a - \hat{a})^2\right]$$

where a is an exogenous technology shock. After receiving a signal they would chose $\hat{a}$ to try to get as close to a as possible. However, this alternative would complicate the solution method significantly. Among others, a new process for technology would need to be estimated and a new signal would add one extra state variable to the problem. Given the difficulties already present in the problem, I have opted for a simplified version where attention to productivity has a perfectly known effect.

as do the basic ideas of the behavioral learning, and robust control literatures." This idea is very appealing for small firms, where the owner is the one taking the decisions and its limited information processing capacity is his own. On the other hand, for big firms - where managers can hire analysts - this limited capacity is less likely to hold. A further extension could be to add "a market for attention," where the limited resources in the country refer to the limited availability of analysts.

Regarding the information structure, I will assume that firms receive a signal of the monetary shock which is more precise the more attention they pay to the aggregate variables

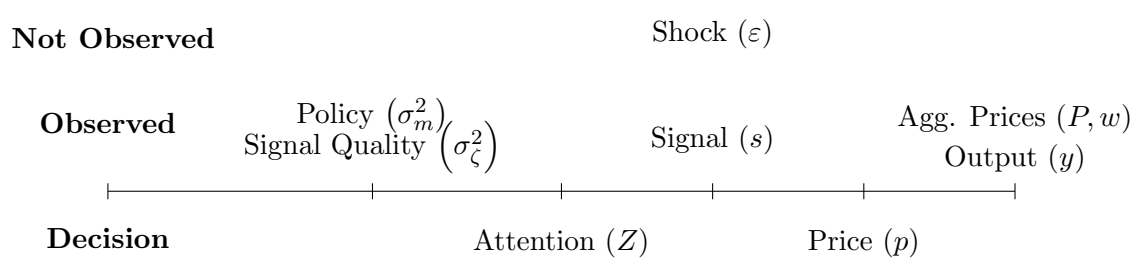
$$s_i = \varepsilon + \zeta_i, \tag{10}$$

where ζ_i are independent of A_i and M, and across firms. Finally, ζ_i is Gaussian white noise with variance

$$\sigma_\zeta^2 (1 - T_i)^\tau = \sigma_\zeta^2 Z_i^\tau. \tag{11}$$

Notice that as attention to productivity (Z) increases so does the noise in the signal.

A timeline of the sequence of events for the firms would be



At the beginning of the period firms observe the characteristics of the economy - including the monetary policy, σ_m^2, and the signal quality, σ_ζ^2. Then, they choose an attention level, Z, which after the monetary shock occurs returns them a signal with a quantity of noise that is negatively related to the attention level. Having observed the signal, and knowing its quality from the choice of attention, the firm chooses a price level. After all firms go through this process, the aggregate price index and wage are realized, leading to the aggregate level of consumption and an output level (and profits) for each firm.

Summarizing, the firms' problem is divided in two stages. Following backwards induction, the second stage, where firms choose a price level $p(s; Z)$ given their previous choice of attention Z, the signal

s and the wage w, is:

$$V(s;Z) = \max_{p(s;Z)} \mathbb{E}_{\varepsilon|s,Z}\left[\left(\frac{p(s;w,Z)}{P}y - \frac{w}{P}l\right)\right], \tag{12}$$

subject to the households' goods demand

$$y = c = \left(\frac{P}{p(s;w,Z)}\right)^{\theta} C, \tag{13}$$

the households' money demand

$$\frac{M}{P} = \gamma_m C, \tag{14}$$

the production function (7), aggregate price (4), the money supply (5)-(6) and the firm's signal which is distributed as defined by (6),(10) and (11), for the previously chosen attention level Z. Appendix B shows the first order conditions of this problem.

Notice that since each firm's price depends on its signal, which is unknown to every other firm, the aggregate consumption, the aggregate price and wage are unknown to all firms. This uncertainty, on top of the money supply uncertainty, implies that each firm does not know with certainty how much it will sell when it chooses its price level. Taking this into account, on the first stage, firms choose an attention level in order to maximize their expected profit, bearing in mind that although paying attention to the aggregate conditions reduces the uncertainty it also reduces the productivity

$$\max_{Z} \int V(s;w,Z)\, f(s|Z)\, ds, \tag{15}$$

subject to the productivity level (8), where $f(s|Z)$ is the distribution of signals given the attention leven as defined by (6), (10) and (11).

Methodology

Given the non-linearity of the expectations and the non-trivial first order condition with respect to the attention level Z, I have solved the problem numerically. In order to determine their output level, firms have to predict elements that depend on the aggregate price level. Since this depends

on how other firms will respond to their signals, I will assume that the wage follows a function that depends on the shock and check that this is true in equilibrium (following rational expectations)[11]

$$w = F_w(\varepsilon). \tag{16}$$

Optimally using the signal, s_i, to predict the monetary shock, ε, implies that, combining (6), (10) and (11),

$$\varepsilon|s \sim N\left(\frac{\left(\sigma_\zeta^2 Z_i^\tau\right)^{-1} s - \sigma_m^{-2}\frac{\sigma_m^2}{2}}{\left(\sigma_\zeta^2 Z_i^\tau\right)^{-1} + \sigma_m^{-2}}, \left[\left(\sigma_\zeta^2 Z_i^\tau\right)^{-1} + \sigma_m^{-2}\right]^{-1}\right).$$

Then, using the wage function (16), firms would use the signals to predict functions of the aggregate consumption (3), aggregate price (4) and money supply (5).

4 Equilibrium and Implications

Definition 1: *Given the monetary shock, ε, an equilibrium for this economy is a set of decision rules for prices, $p(s;Z)$, and for the attention level, Z; labour supply L, money demand M^d, consumption demand c_i and labour demand l_i for all $i \in [0,1]$; and a wage w such that: (i) given the wage and prices, M^d, L and c_i for all $i \in [0,1]$ solve the households' problem; (ii) $p(s;Z)$ and Z solve the firms' problem; (iii) good i market clears, for all $i \in [0,1]$; (iv) the labour market clears*

$$L = \int_0^1 l_i di; \tag{17}$$

and (v) the money market clears

$$M^d = M^s = \bar{M}e^{\varepsilon}. \tag{18}$$

I use an iterative procedure to solve for the rational expectations equilibrium of the model. First, I make a guess concerning the nominal wage. Second, I solve the firms' attention problem (12)-(15). Third, I aggregate the individual prices to obtain the aggregate price (4). Fourth, I compute

[11] The numerical solution is based on guessing an exponential polynomial for the wage. Given this, I numerically solve for the attention level and individual price. Then, I simulate and check that the guess on the wage function was correct and, if not, update the guess until it is. See Appendix C for details.

the individual consumption level and the aggregate one (3). Fifth, I compute the aggregate labour demand and supply implied, and check if the market clears. If it does not, I update the guess until the market does clear. Appendix C explains the exact procedure and the computational tools used to solve this problem.

Then, in order to analyze the implications of the model, I have checked the results for some parameters values common in the literature.[12] First, in this model money is neutral only when there is no uncertainty about the monetary shock. This happens in 3 main cases: (i) monetary policy is fixed ($\sigma_m^2 = 0$); (ii) there is no noise in the signal ($\sigma_\zeta^2 = 0$); or (iii) full attention is paid to aggregate conditions ($Z = 0$). While the first two reasons are exogenous, the last one is endogenously determined.

Second, as expected, attention level to productivity decreases with monetary uncertainty (its opportunity cost) and increases with its marginal return (η). For very low monetary uncertainty, full attention is paid to productivity. As it increases, the attention to productivity (macro uncertainty) decreases (increases) monotonically. Regarding the noise of the signal (σ_ζ), the noisier it is, the more attention is put to it so that its noise is reduced (as long as the monetary policy is uncertain enough), since its return in noise reduction is bigger. The output level follows the same pattern: The higher the uncertainty, the lower the productivity and the lower the aggregate output. Figure 1 shows the before-mentioned relationship between monetary volatility and attention allocation and aggregate output. The attention level to productivity and the aggregate output increase with the marginal return (η) of attention to productivity. Finally, all these effects also cause that the expected firm profits decrease with both the monetary uncertainty and the noise in the signal.

Figure 2 shows the policy function for two different amounts of monetary uncertainty. As uncertainty increases, more attention is paid to the signal, making it much more reliable. Under such a scenario, the policy function becomes more sensitive to the signal since firms trust it more. On the other hand, in cases of low uncertainty, the monetary shock is easily predictable without the signal, which is also mainly noise. Therefore, the policy function is flatter and relatively less sensitive to the signal.

Finally, Figure 3 shows the relationship between the rest of the aggregate variables and the monetary

[12]For the utility function, $\gamma_l = 0.94, \gamma_m = 1, \Psi = 3,\ \theta = 4$ as suggested by Chetty (2006) and by the typical Cash-in-Advance model. Regarding the production function, $\alpha = 0.8$ is a commonly used value and, normalizing, $\bar{A} = 1$ is used. Finally, $M = 1$ is used but this only has an effect in the wage and prices levels. For the parameters to be calibrated, $\eta = 0.16$ and $\tau = 2.97$ are used since these are the estimated values from Section 5. Finally, the monetary uncertainty $\sigma_m^2 \in [0.01, 2]$ is used and the signal quality σ_ζ^2 is assumed to be equal to the monetary uncertainty.

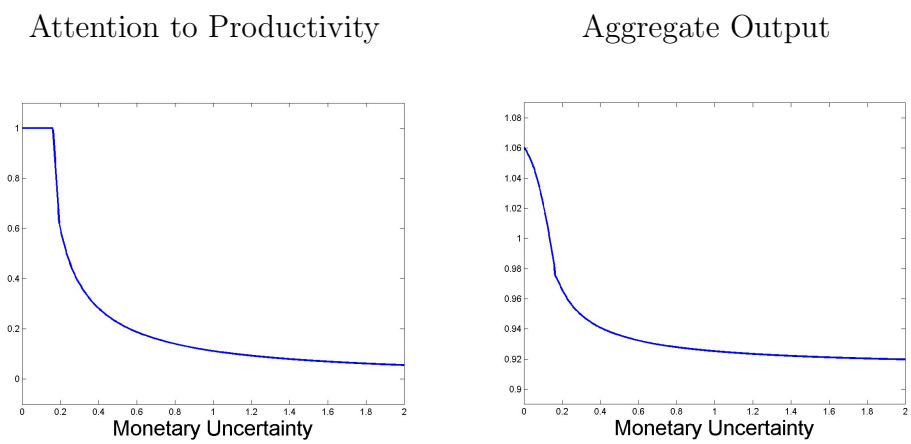
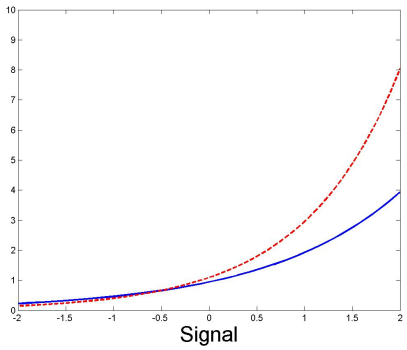


Figure 1: Attention and aggregate output and monetary uncertainty.



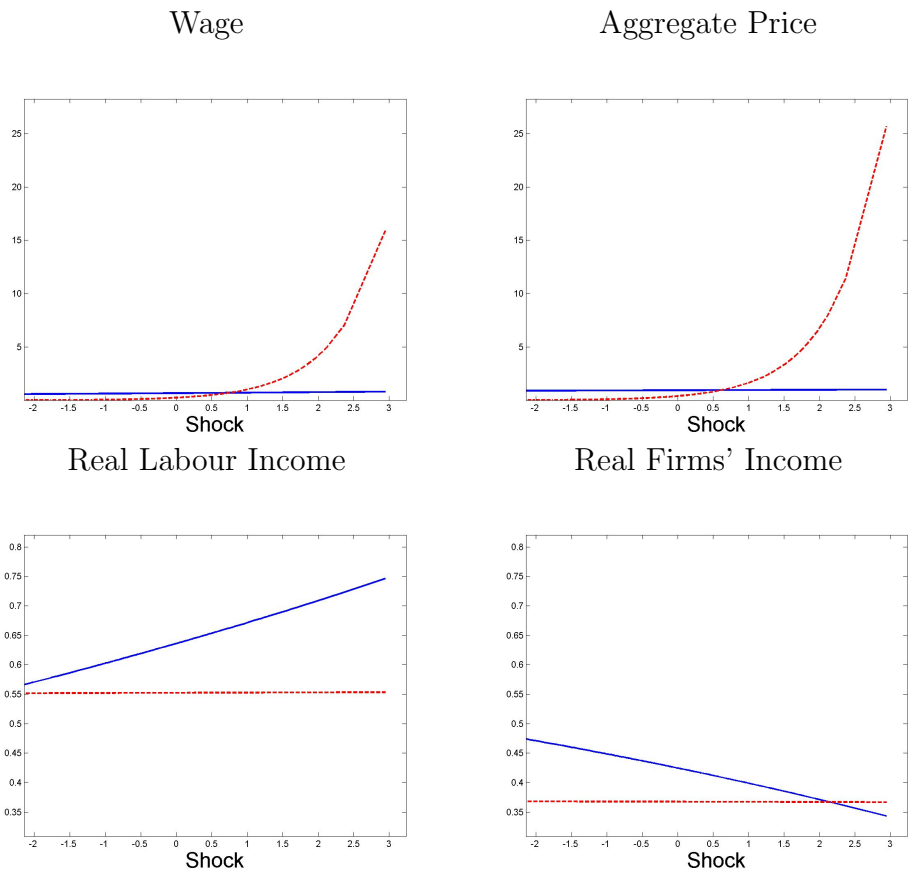
Low uncertainty (Blue, solid) and high uncertainty (Red, dashed).

Figure 2: Policy Functions $P(s_i, w)$

shock in environments of low and high uncertainty.[13] Aggregate prices and nominal wages increase with a positive monetary shock, in such a way that real income is redistributed towards labour. On the other hand, under high uncertainty (red, dashed), shows that this model would predict that aggregate prices and nominal wages increase with a positive monetary shock once again, but the distribution of real income remains stable since firms pay high attention to the aggregate uncertainty, and end up almost eliminating it.[14]

[13] In order to standardize the shock, the x-axis actually refers to the shock minus its mean divided by its standard deviation. Otherwise, an economy with high uncertainty could have a very high shock that is not actually "surprising."

[14] This resembles the third case of money neutrality mentioned earlier in this section.



Low uncertainty (Blue, solid) and high uncertainty (Red, dashed).

Figure 3: Wages, Prices, Real Income and Monetary Shocks.

5 Quantitative Analysis

In this section I perform a simple calibration strategy that takes the values for previously studied parameters from the literature and assigns values to the remaining three novel parameters by assuming one of them (and doing some robustness checks) and targeting two moments on the output and monetary uncertainty for Ecuador. Then, using these values, I study how much of the output cycle can be explained by this model and evaluate the consumption cost of monetary uncertainty.

Calibration

Since the model requires the monetary policy uncertainty as an input, I first fit a modified (so that it allows for a shock with the mean the model assumes) generalized auto-regressive conditional heteroskedasticity (GARCH (1,1)) model on $g_{m,t}$, the money (M2) growth at period t, for t between 1960 and 2010

$$g_{m,t} = \frac{\sigma_{m,t}^2}{2} + \varepsilon_t$$

where the shock follows the same structure as (6)

$$\varepsilon_t \sim N\left(-\frac{\sigma_{m,t}^2}{2}, \sigma_{m,t}^2\right).$$

and

$$\sigma_{m,t}^2 = c + \beta_1\left(\varepsilon_{t-1}^2 - \sigma_m^2\right) + \beta_2\left(\sigma_{m,t-1}^2 - \sigma_m^2\right), \tag{19}$$

$$\sigma_m^2 = \frac{c}{1-\beta_1-\beta_2}, \tag{20}$$

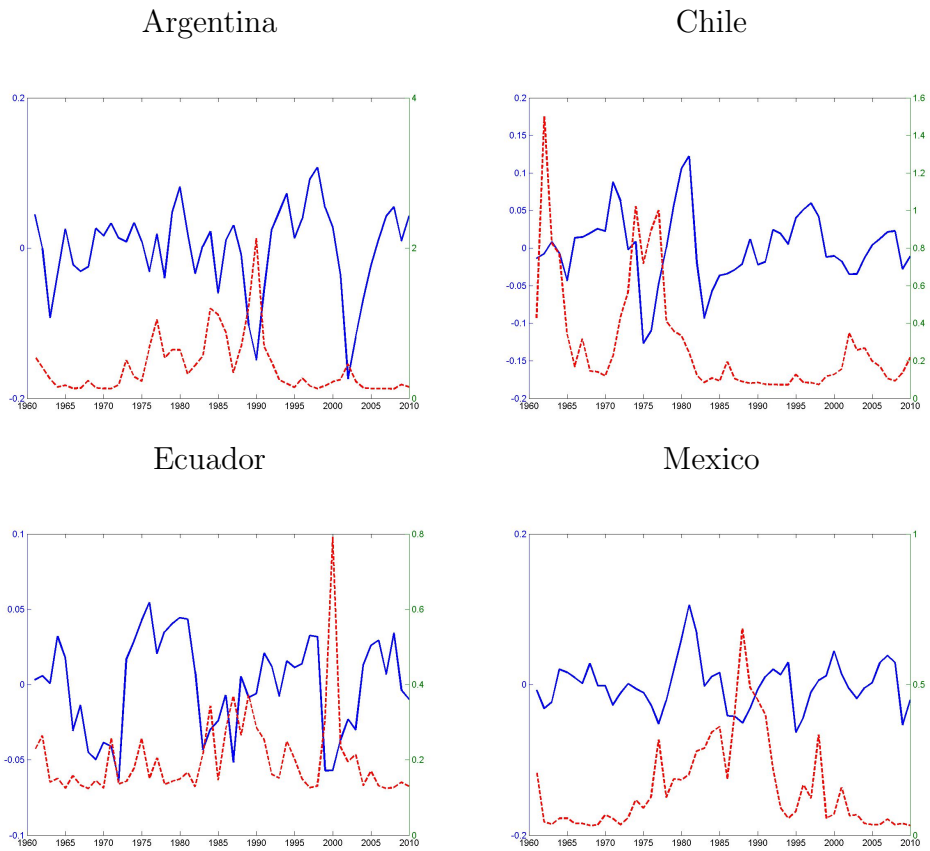
which estimates the monetary uncertainty taking into account a possible correlation across time.[15] This also provides an estimation of the monetary shock which is used in the calibration and the analysis.[16] Figure 4 shows the calculated level of monetary uncertainty as well as the HP-filtered output cycle for the four countries of study: Argentina, Chile, Ecuador and Mexico. It is important to remark that the scale is different for each country since, for example, Argentina shows a significantly more unstable monetary policy. As the figure implies, there is negative correlation between the output cycle and the monetary policy uncertainty which is shown in Table 3.

Secondly, the model also requires an estimation of the noise present in the signal. Since it is reasonable to think that the variation in the noise present in the signal also increases with uncertainty (otherwise, the noise becomes irrelevant for high levels of uncertainty), I assume that

$$\sigma_{\zeta,t}^2 = k\sigma_{m,t}^2, \tag{21}$$

[15] This is an extension to the model Baillie *et al* (1996) applied to inflation uncertainty, which allows for a shock with a mean different from zero.

[16] I fit the GARCH model on the money growth directly, and not on the HP-filtered cycles, since this strategy avoids problems with the timing of the information on the monetary policy available to the firms. Applying an HP filter has the problem that it uses information on future periods (since it is a double-sided filter) which would not necessarily be available to the firms. Therefore, it would eliminate part of the volatility by assigning it to this trend which was built on more information than the available one at each point in time. As a robustness check I have also estimated the model after applying the filter to the monetary policy. The results are qualitatively similar although the fit does seem to worsen.



Output Cycle (Solid, left axis) and Monetary Std. Dev. (Dashed, right axis)

Figure 4: Output Cycle and Monetary Uncertainty

Table 3: Monetary Policy and GDP.

	Std.Dev(GDP)	**Corr(Uncertainty,GDP)**
Argentina	0.058	-0.400
Chile	0.047	-0.221
Ecuador	0.032	-0.385
Mexico	0.033	-0.144

making the noise-signal ratio constant.[17]

I calibrate two parameters: (i) the productivity return, η, as defined by (8); and (ii) the non-linearity of the uncertainty reduction from attention, τ, as defined by (11). The noise-signal ratio, k, from (21), is assumed to take the value of one. In other words, the noise increases one to one with the monetary uncertainty.[18] The rest of the parameters have already been studied in the literature and are taken as given since the calibration of these parameters is already very time consuming.[19] The empirical targets in order to calibrate the mentioned parameters is the standard deviation of HP-filtered output cycle and the correlation between output cycle and monetary uncertainty for Ecuador.[20] Table 4 shows the result from this calibration

Analysis

Using the parameters values from Table 4 and calculating the monetary policy uncertainty as in (19)-(20), I now study how much the model can explain of the output cycle observed in the four Latin-American countries mentioned before. In order to establish this capacity of the model, I

[17]It is constant without taking into account the effect from the attention allocation decision.

[18]This parameter could be calibrated as well but as of now the calibration is very time consuming so I have adopted this alternative. For robustness purposes, I have also calibrated the model taking $\kappa = 0.5$ and $\kappa = 2$ and found that the results remain qualitatively and quantitatively similar (but the two parameters calibrated do change).

[19]The calibration, although coded in C++, required approximately 76 hours.

[20]I chose Ecuador because it exhibits a fluctuating monetary policy: fairly stable until 1983, when it went through a major devaluation; a crawling peg potentially causing constant devaluations; foreign exchange value crisis in 1999 followed by a dollarization. Moreover, regarding the data estimation it is in one of the few countries to clearly exhibit a positive correlation between output cycle and monetary shocks, which was always found true when solving the model. This suggests that a better methodology to estimate the monetary policy in the data might be necessary to differentiate monetary volatility from shocks (Fernandez-Villaverde and Rubio-Ramirez, 2010)

Table 4: Parameters values

Calibrated		
η	Productivity return	0.160
τ	Non-linearity of noise reduction	2.971
Assumed + Robustness Check		
k	Noise-signal ratio	1
Obtained from Literature		
γ_l	Utility multiplier of leisure	0.94
γ_m	Utility multiplier of real money	1
Ψ	Utility leisure Non-linearity	3
θ	Consumption Aggregation	4
α	Production Function Non-linearity	0.8

simulate the model using the monetary policy estimated for each country and then perform the same transformation done to the real data obtained from each country.[21] Figure 5 shows the result from this comparison. I also apply the calibrated results to other Latin-American countries: Argentina, Chile and Mexico. This can be seen in Figure 6. Table 5 shows the model results for the same moments calculated in the data and shown in table 3. The fit is not as good as for Ecuador, but it is important to recall that no data from those countries was used in the calibration. With this caveat in mind, it is clear that the model predicts a higher absolute correlation between monetary uncertainty and output than the data shows, particularly for countries with lower correlation like Mexico and Chile. An alternative to improve this fit would be to calibrate each countries parameters separately, but I have abstained from this due to computational reasons.[22]

Table 5: Model Moments.

	Std.Dev(GDP)	**Corr(Uncertainty,GDP)**
Ecuador	0.037	-0.404
Argentina	0.041	-0.283
Chile	0.041	-0.485
Mexico	0.044	-0.514

A quantitative proxy for the capacity of the model to explain these fluctuations can be performed by dividing the model output by the one from the data,[23] and averaging across the whole time series.

[21] Notice that since the model is static, each year is run independently from each other.

[22] I have performed the exercise of calibrating the model with Argentinean data instead, but this is neither used nor shown here. However, the results are similar to the ones shown here.

[23] Considering that if this is above one a value of one is set and, similarly, when it is found negative a value

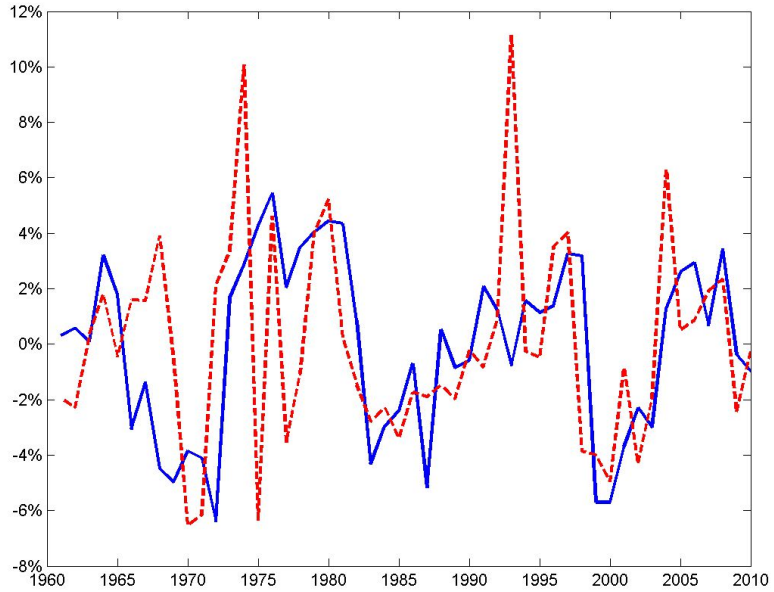


Figure 5: Ecuador, Data (Solid) and Model (Dashed) comparison.

Table 6 shows the results from such study for each country. Focusing on the country used for the calibration, Ecuador, the model can explain approximately 48.7% of the output cycle fluctuations. Moreover, the correlation between the model and the data is 35.6%. Probably due to the calibration design, when comparing all countries, the best fit is observed for Ecuador. Interestingly, major events in Ecuador's recent monetary history are well captured by the model. Both the 1983 devaluation together with the banking 1999 crisis (where money started growing at a rate of 170% to pay back depositors) followed by the recovery after the dollariztion in Ecuador are almost perfectly fit. On the opposite end, the model fit for Mexico is the poorest of the four, probably because its cycles are less related to monetary policy issues (Garriga, 2010).

In order to evaluate how important the uncertainty itself is for the fit of the model, it is useful to test the model with the observed monetary uncertainty but with the expected shocks instead of the estimated ones. Table 7 shows that the fit worsens for Ecuador but seems to improve for Argentina and Mexico. A potential reason for this improvement is, as suggested before, that these countries do not show a clear positive correlation between the monetary shock and output, which holds true for the model. Hence, shocks do seem to matter on top of uncertainty for the model fit.

of zero is set.

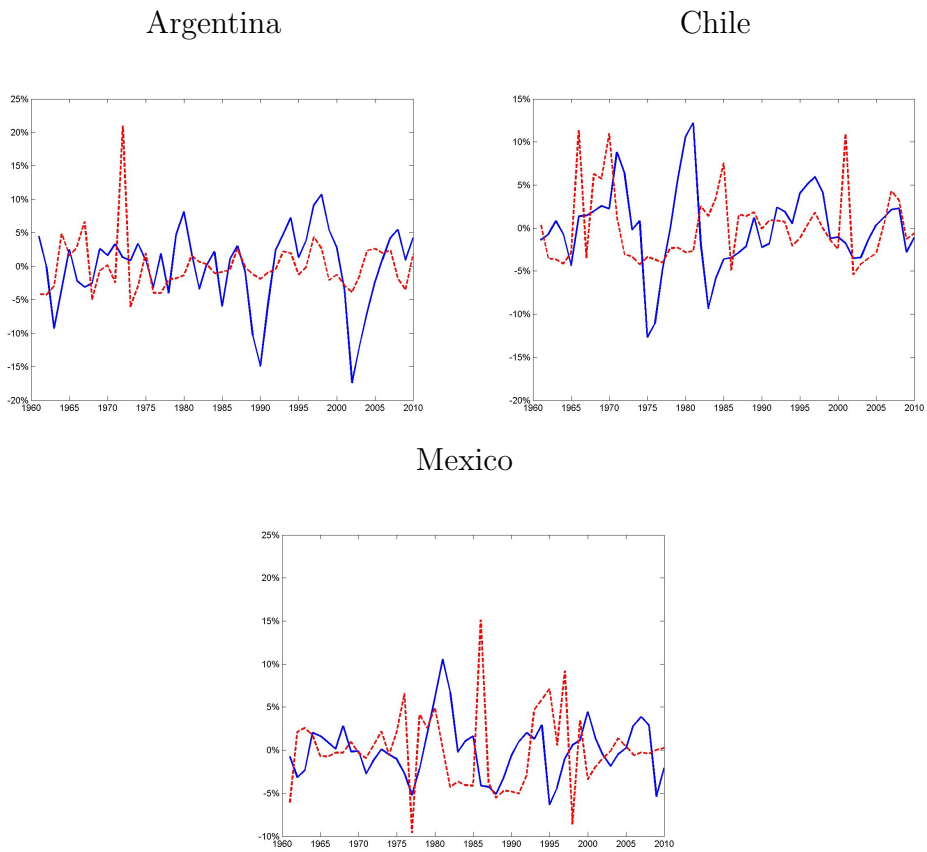


Figure 6: Data (Solid) and Model (Dashed) comparison.

Table 6: Model Capacity.

	Correlation	**Explains**
Ecuador	0.356	48.7%
Argentina	0.167	28.9%
Chile	0.094	43.3%
Mexico	-0.075	35.9%
Average	0.135	39.2%

Last but not least, probably the most important analysis to be performed with this model is the study of the cost of monetary uncertainty in terms of consumption. For this I study again the effect of monetary uncertainty using the expected shocks (therefore showing only the effect of the uncertainty

Table 7: Model Capacity (with expected monetary shocks).

	Correlation	**Explains**
Ecuador	0.331	41.6%
Argentina	0.421	27.1%
Chile	0.087	38.5%
Mexico	0.062	30.0%
Average	0.225	34.3%

itself) and compare the difference in output with respect to a situation with low uncertainty. Taking the magnitude of monetary uncertainty observed in each country and the corresponding expected shock, Figure 7 shows how costly monetary uncertainty has been for Ecuador in terms of consumption each year.

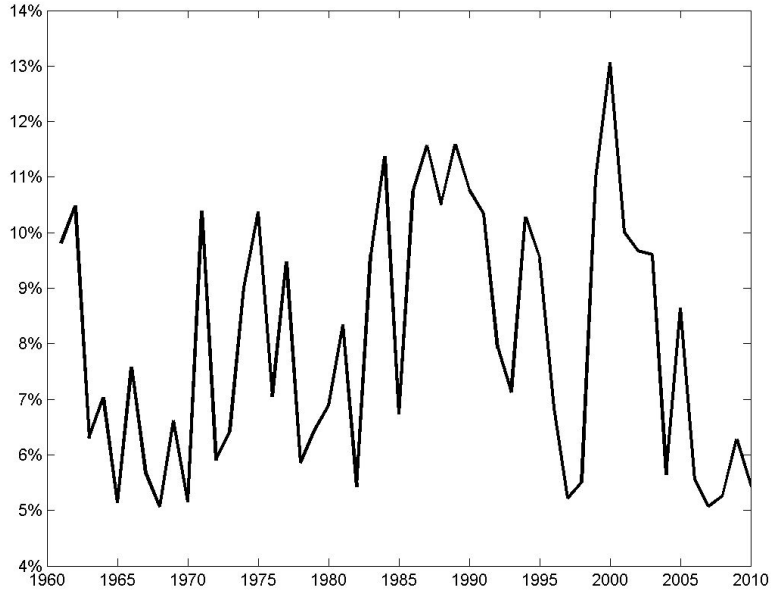


Figure 7: Ecuador's consumption losses from monetary uncertainty.

Table 8 shows the maximum and average cost observed for each country. Even though the average seems to be the most relevant summarizing cost, it is important to remark that the maximum loss is very significant when borrowing constraints are present since households cannot smooth their consumption. Focusing on Ecuador, the results suggest that monetary uncertainty can generate

losses in consumption of up to 13% in only one year and annual average losses as high as 8%. If we included the other countries, the model would suggest the costs could be even higher, as seen in the Argentinean case.

Table 8: Monetary Uncertainty cost in Consumption.

	Average	**Maximum**
Ecuador	8.0%	13.1%
Argentina	9.9%	13.9%
Chile	7.6%	13.7%
Mexico	5.1%	12.9%

6 Conclusion and Comments

Worldwide data suggests that policy (including monetary) uncertainty is negatively correlated with growth. Many models provide a rational for this through investment. However, to the best of my knowledge, there has been no research on the effects of monetary uncertainty and attention allocation, which through a simple scarce resource argument could lead to reduction in attention paid to productivity. This paper tries to fill this gap, by presenting a static model of an economy where firms have to allocate attention in order to reduce the uncertainty on an aggregate monetary shock, with a cost in terms of productivity. This provides a rationale for the relationship observed between monetary volatility and aggregate output in Latin-American countries.[24] Money is neutral when there is no uncertainty about the shock, which can be due to either endogenous or exogenous conditions which are explained in Section 4. Similarly to the intuition in the Rational Inattention literature, as uncertainty increases, attention is drawn towards its reduction. However, in this model, this has a cost in terms of productivity, leading to a reduction in total output.

Applying the model to Latin-American countries, where monetary uncertainty is a major source of policy uncertainty, I find that the model is capable of explaining approximately 48% of the fluctuations observed. Probably more importantly for policy analysis, the model suggests that monetary uncertainty can be very costly in terms of consumption since it can generate consumption losses of up to 13% in only one year, with an average of around 8%. It should be remarked that this is significantly bigger than what has been found in the previous literature on the effects of stabilization

[24]Note that the model could be easily modified to allow for other sources of uncertainty that could be more relevant for other regions.

on the level of consumption as described in Section 2.

Finally, as previously suggested, many extensions could and should be developed in this model in order to fully exploit its capacity and perform an improved policy analysis. Some relatively minor extensions that could provide important benefits would be: (i) to allow for heterogeneous technologies across firms;[25] and (ii) to allow for a "market for attention," where firms can hire analysts - which should be a limited resource in the economy - to reduce the uncertainty on aggregate conditions.[26] A major extension that could provide very interesting policy analysis and allow for comparisons in long-term growth rates across countries would be to transform the model into one of growth, where the attention on productivity affects its growth rate instead of its current level only. This has the potential to explain growth trend differences between developed and developing countries. Finally, an interesting extension to see whether the attention channel is important when studying policy uncertainty would be to include the more standard channel of investments. Such a model would provide the means to test the relative importance of each channel, which could be vary across countries.

[25]This would provide more difficulties in the computation of the solution since expectations should now consider two extra sources of variation: the current technology level and the attention level for each firm.

[26]This would be a very appealing extension for the interpretation of the model for big firms, where managers do usually hire analysts to predict current economic conditions. However, this would be problematic in the estimation of the availability of these analysts in the economy.

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Appendix

Appendix A: Households' problem solution

The problem is the following, where individuals are assumed to observe money and prices perfectly.

$$\max_{c_i,L,M} U + \lambda \left[WL + D - \int_0^1 p_i c_i di - M \right]$$

where

$$U = \ln(C) + \gamma_m \ln\left(\frac{M}{P}\right) - \gamma_l \frac{L^{1+\Psi}}{1+\Psi}$$

$$C = \left[\int_0^1 (c_i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

Then, the first order condition with respect to consumption, c_i, is:

$$C^{-1} \left(\int_0^1 (c_i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{1}{\theta-1}} c_i^{-\frac{1}{\theta}} - \lambda p_i = 0$$

which leads to

$$C^{\frac{1}{\theta}-1} c_i^{-\frac{1}{\theta}} = \lambda p_i \tag{22}$$

and

$$\lambda = \frac{C^{-1}}{P} \tag{23}$$

Then, using (23) in (22) we get consumption demand for each variety as a function of the aggregate price and demand

$$c_i = \left(\frac{P}{p_i}\right)^{\theta} C$$

The first order condition with respect to labour, L, is

$$-\gamma L^{\psi} + \lambda W = 0$$

and using (23) leads to

$$\frac{W}{P} = \gamma_l L^{\psi} C$$

Finally, the first order condition with respect to money demand, M, combined with (23) is

$$\frac{M}{P} = \gamma_m C$$

Appendix B: The attention problem

The expected profit in the second stage as defined by (12) is

$$E\Pi = E\left[\left(\frac{p}{P}\right)^{1-\theta}\frac{M}{P}\frac{1}{\gamma_m} - \frac{W}{P}\left[\bar{A}\left(1+\eta Z_i\right)\right]^{-\frac{1}{\alpha}}\left[\left(\frac{P}{p_i}\right)^{\theta}\frac{M}{P}\frac{1}{\gamma_m}\right]^{\frac{1}{\alpha}}\right]$$

which has a first order condition with respect to the price level, for a given attention level Z, that is equal to:

$$(1-\theta)\,p^{-\theta}\frac{1}{\gamma_m}E_s\left[MP^{\theta-2}|s,Z\right] + \frac{\theta}{\alpha}p^{-\frac{\theta+\alpha}{\alpha}}\left(\frac{1}{\gamma_m}\right)^{\frac{1}{\alpha}}A^{-\frac{1}{\alpha}}E_s\left[WM^{\frac{1}{\alpha}}P^{\frac{\theta-\alpha-1}{\alpha}}|s,Z\right] = 0$$

Therefore, the optimal price is

$$p^* = \left[\frac{\theta}{\theta-1}\frac{1}{\alpha}\left[\bar{A}\left(1+\eta Z_i\right)\right]^{-\frac{1}{\alpha}}\left(\frac{1}{\gamma_m}\right)^{\frac{1-\alpha}{\alpha}}\frac{E_s\left[WM^{\frac{1}{\alpha}}P^{\frac{\theta-\alpha-1}{\alpha}}|s,Z\right]}{E_s\left[MP^{\theta-2}|s,Z\right]}\right]^{\frac{\alpha}{(\theta-1)(1-\alpha)+1}} \tag{24}$$

In order to proceed with the first stage of the problem, we guess that the aggregate wage and price are exponential polynomials

$$W = \exp\left(A_w + B_w\varepsilon\right) \tag{25}$$

$$P = \exp\left(A_p + B_p\varepsilon\right) \tag{26}$$

so the two elements in the pricing function (24) become

$$P^{\frac{\theta-\alpha-1}{\alpha}} = \exp\left(A_{p1} + B_{p1}\varepsilon\right)$$

$$P^{\theta-2} = \exp\left(A_{p2} + B_{p2}\varepsilon\right)$$

where I have defined

$$\begin{aligned}
A_{p1} =& \frac{(\theta-\alpha-1)\,A_p}{\alpha} \\
B_{p1} =& \frac{(\theta-\alpha-1)\,B_p}{\alpha} \\
A_{p2} =& (\theta-2)\,A_p \\
A_{p1} =& (\theta-2)\,B_p
\end{aligned}$$

and recall that

$$M^{\frac{1}{\alpha}} = \bar{M}^{\frac{1}{\alpha}} \exp\left(\frac{\varepsilon}{\alpha}\right).$$

Also recall that the condition distribution of the monetary shock is

$$\varepsilon|s \sim N\left(\frac{\left(\sigma_\zeta^2 Z_i^\tau\right)^{-1} s - \sigma_m^{-2}\frac{\sigma_m^2}{2}}{\left(\sigma_\zeta^2 Z_i^\tau\right)^{-1} + \sigma_m^{-2}}, \left[\left(\sigma_\zeta^2 Z_i^\tau\right)^{-1} + \sigma_m^{-2}\right]^{-1}\right)$$

and under the assumption that $\sigma_\zeta^2 = k\sigma_m^2$, this simplifies to

$$\varepsilon|s \sim N\left(\frac{(kZ_i^\tau)^{-1} s - \frac{\sigma_m^2}{2}}{(kZ_i^\tau)^{-1}+1}, \frac{\sigma_m^2}{(kZ_i^\tau)^{-1}+1}\right). \tag{27}$$

So the two expected elements in the pricing equation (24) simplify to

$$\begin{aligned}
E_s\left[WM^{\frac{1}{\alpha}}P^{\frac{\theta-\alpha-1}{\alpha}}|s,Z\right] =& \bar{M}^{\frac{1}{\alpha}} \exp\left(\begin{array}{c} A_w + A_{p1} + \\ +\frac{\left[\frac{1}{\alpha}+B_w+B_{p1}\right]}{\left(kZ_i^\tau\right)^{-1}+1}\left[(kZ_i^\tau)^{-1} s + \left[\frac{1}{\alpha}+B_w+B_{p1}-1\right]\frac{\sigma_m^2}{2}\right] \end{array}\right) \\
E\left[MP^{\theta-2}|s,Z\right] =& \bar{M}\exp\left(A_{p2} + \frac{[1+B_{p2}]}{(kZ_i^\tau)^{-1}+1}\left[(kZ_i^\tau)^{-1} s + B_{p2}\frac{\sigma_m^2}{2}\right]\right)
\end{aligned}$$

Combining the two, the ratio simplifies to

$$\frac{E_s\left[WM^{\frac{1}{\alpha}}P^{\frac{\theta-\alpha-1}{\alpha}}|s,Z\right]}{E\left[MP^{\theta-2}|s,Z\right]} = \bar{M}^{\frac{1-\alpha}{\alpha}} \exp\left(\begin{array}{c} A_w + A_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right) \\ +\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]\frac{\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1}s \\ +\frac{\left\{\left[\frac{1}{\alpha}+B_w+B_{p1}\right]\left[\frac{1}{\alpha}+B_w+B_{p1}-1\right]-[1+B_{p2}]B_{p2}\right\}\frac{\sigma_m^2}{2}}{\left(kZ_i^\tau\right)^{-1}+1} \end{array}\right)$$

Using this we can now rewrite the price (24) as

$$p^* = \exp\left(a_p + b_p s\right)$$

where

$$a_p = \left[\begin{array}{c} \frac{\alpha}{(\theta-1)(1-\alpha)+1}\log\left(\frac{\theta}{\theta-1}\frac{1}{\alpha}\left[\bar{A}\left(1+\eta Z_i\right)\right]^{-\frac{1}{\alpha}}\left(\frac{\bar{M}}{\gamma_m}\right)^{\frac{1-\alpha}{\alpha}}\right) \\ +\frac{\alpha}{(\theta-1)(1-\alpha)+1}\left(A_w + A_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right)+ \\ +\frac{\alpha}{(\theta-1)(1-\alpha)+1}\frac{\left\{\left[\frac{1}{\alpha}+B_w+B_{p1}\right]\left[\frac{1}{\alpha}+B_w+B_{p1}-1\right]-\left[1+B_{p2}\right]B_{p2}\right\}\frac{\sigma_m^2}{2}}{\left(kZ_i^\tau\right)^{-1}+1} \end{array}\right] \tag{28}$$

$$b_p = \frac{\alpha}{(\theta-1)(1-\alpha)+1}\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]\frac{\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1} \tag{29}$$

This new way of writing the price allows us to obtain the aggregate price,

$$P = \left[\int p_i^{1-\theta}di\right]^{\frac{1}{1-\theta}} = \exp\left(a_p\right)\left[\int \exp\left(\left(1-\theta\right)b_p s_i\right)di\right]^{\frac{1}{1-\theta}}$$

and (using $s|\varepsilon \sim N\left(\varepsilon, kZ_i^\tau\sigma_m^2\right)$) that integral is equal to

$$\int \exp\left(\left(1-\theta\right)b_p s_i\right)di = \exp\left[\begin{array}{c} \frac{\alpha(1-\theta)}{(\theta-1)(1-\alpha)+1}\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]\frac{\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1}\varepsilon+ \\ +\left(\frac{\alpha(1-\theta)}{(\theta-1)(1-\alpha)+1}\right)^2\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]^2\frac{\left(kZ_i^\tau\right)^{-1}}{2\left[\left(kZ_i^\tau\right)^{-1}+1\right]^2}\sigma_m^2 \end{array}\right]$$

which leads to an aggregate price equal to

$$P = \exp\left[\begin{array}{c} a_p+ \\ +\left(1-\theta\right)\left(\frac{\alpha}{(\theta-1)(1-\alpha)+1}\right)^2\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]^2\frac{\left(kZ_i^\tau\right)^{-1}}{2\left[\left(kZ_i^\tau\right)^{-1}+1\right]^2}\sigma_m^2 \end{array}\right]\times$$
$$\times\exp\left[b_p\varepsilon\right]$$

So for our guess $P = \exp\left(A_p + B_p\varepsilon\right)$ to work it must be that

$$A_p = \left[\begin{array}{c} \frac{\alpha}{(\theta-1)(1-\alpha)+1}\log\left(\frac{\theta}{\theta-1}\frac{1}{\alpha}\left[\bar{A}\left(1+\eta Z_i\right)\right]^{-\frac{1}{\alpha}}\left(\frac{\bar{M}}{\gamma_m}\right)^{\frac{1-\alpha}{\alpha}}\right) \\ +\frac{\alpha}{(\theta-1)(1-\alpha)+1}\left(A_w + A_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right)+ \\ +\frac{\alpha}{(\theta-1)(1-\alpha)+1}\frac{\left\{\left[\frac{1}{\alpha}+B_w+B_{p1}\right]\left[\frac{1}{\alpha}+B_w+B_{p1}-1\right]-\left[1+B_{p2}\right]B_{p2}\right\}\frac{\sigma_m^2}{2}}{\left(kZ_i^\tau\right)^{-1}+1}+ \\ +\left(1-\theta\right)\left(\frac{\alpha}{(\theta-1)(1-\alpha)+1}\right)^2\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]^2\frac{\left(kZ_i^\tau\right)^{-1}}{2\left[\left(kZ_i^\tau\right)^{-1}+1\right]^2}\sigma_m^2 \end{array}\right]$$

and

$$B_p = \frac{\alpha}{(\theta-1)(1-\alpha)+1}\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]\frac{(kZ_i^\tau)^{-1}}{(kZ_i^\tau)^{-1}+1}$$

These imply that

$$A_p = \left[\begin{array}{c} \alpha\log\left(\frac{\theta}{\theta-1}\frac{1}{\alpha}\left[\bar{A}(1+\eta Z_i)\right]^{-\frac{1}{\alpha}}\left(\frac{\bar{M}}{\gamma_m}\right)^{\frac{1-\alpha}{\alpha}}\right)+\alpha A_w+ \\ +\alpha\frac{\left\{\left[\frac{1}{\alpha}+B_w+B_{p1}\right]\left[\frac{1}{\alpha}+B_w+B_{p1}-1\right]-[1+B_{p2}]B_{p2}\right\}\frac{\sigma_m^2}{2}}{\left(kZ_i^\tau\right)^{-1}+1}+ \\ +(1-\theta)\left(\frac{\alpha^2}{(\theta-1)(1-\alpha)+1}\right)\left[\frac{1}{\alpha}-1+B_w+B_p\left(\frac{(\theta-1)(1-\alpha)}{\alpha}\right)\right]^2\frac{\left(kZ_i^\tau\right)^{-1}}{2\left[\left(kZ_i^\tau\right)^{-1}+1\right]^2}\sigma_m^2 \end{array}\right]$$

$$B_p = \frac{\alpha}{(\theta-1)(1-\alpha)+1+(kZ_i^\tau)^{-1}}\left[\frac{1}{\alpha}-1+B_w\right](kZ_i^\tau)^{-1}$$

Then, using (27), the expected profits are

$$\begin{aligned}
\mathbb{E}\Pi_{|s} =& \mathbb{E}\left[\begin{array}{c} \exp\left((1-\theta)(a_p+b_p s)\right)\frac{\bar{M}\exp(\varepsilon)}{\exp((2-\theta)(A_p+B_p\varepsilon))}\frac{1}{\gamma_m}+ \\ -\frac{\exp(A_w+B_w\varepsilon)}{\exp(A_p+B_p\varepsilon)}\left(\bar{A}(1+\eta Z_i)\gamma_m\right)^{-\frac{1}{\alpha}}\frac{\bar{M}^{\frac{1}{\alpha}}\exp\left(\frac{1}{\alpha}\varepsilon\right)}{\exp\left(\frac{(1-\theta)}{\alpha}(A_p+B_p\varepsilon)\right)}\exp\left(-\frac{\theta}{\alpha}(a_p+b_p s)\right) \end{array}\right] \\
=& \frac{\bar{M}}{\gamma_m}\exp\left((1-\theta)a_p-(2-\theta)A_p-\frac{(2-\theta)B_p\left[1-(2-\theta)B_p\right]\sigma_m^2}{2\left[(kZ_i^\tau)^{-1}+1\right]}\right)\times \\
&\times\exp\left(\left[\frac{\left[1-(2-\theta)B_p\right](kZ_i^\tau)^{-1}}{(kZ_i^\tau)^{-1}+1}+(1-\theta)b_p\right]s\right)+ \\
&-\left(\frac{M}{\bar{A}(1+\eta Z_i)\gamma_m}\right)^{\frac{1}{\alpha}}\exp\left(-\frac{\theta}{\alpha}a_p+A_w-A_p\left(\frac{\alpha+1-\theta}{\alpha}\right)\right)\times \\
&\times\exp\left(\frac{\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}\right)\sigma_m^2}{2\left[(kZ_i^\tau)^{-1}+1\right]}\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}-1\right)\right) \\
&\times\exp\left(\left[\frac{\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}\right)(kZ_i^\tau)^{-1}}{(kZ_i^\tau)^{-1}+1}-\frac{\theta}{\alpha}b_p\right]s\right)
\end{aligned}$$

And taking expectations over the signal $s \sim N\left(-\frac{\sigma_m^2}{2}, (kZ_i^\tau + 1)\,\sigma_m^2\right)$,

$$
\begin{aligned}
\mathbb{E}\Pi =& \frac{\bar{M}}{\gamma_m} \exp\left((1-\theta)\, a_p - (2-\theta)\, A_p - \frac{(2-\theta)\, B_p \left[1-(2-\theta)\, B_p\right] \sigma_m^2}{2\left[(kZ_i^\tau)^{-1}+1\right]} \right) \times \\
& \times \exp\left(-\left[\frac{\left[1-(2-\theta)\, B_p\right](kZ_i^\tau)^{-1}}{(kZ_i^\tau)^{-1}+1} + (1-\theta)\, b_p \right] \frac{\sigma_m^2}{2} \right) \times \\
& \times \exp\left(\frac{\left[\frac{[1-(2-\theta)B_p]\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1} + (1-\theta)\, b_p \right]^2}{2} (kZ_i^\tau + 1)\, \sigma_m^2 \right) + \\
& - \left(\frac{M}{\bar{A}\,(1+\eta Z_i)\,\gamma_m} \right)^{\frac{1}{\alpha}} \exp\left(-\frac{\theta}{\alpha} a_p + A_w - A_p \left(\frac{\alpha+1-\theta}{\alpha} \right) \right) \times \\
& \times \exp\left(\frac{\left(B_w - B_p\left(\frac{\alpha+1-\theta}{\alpha}\right) + \frac{1}{\alpha}\right)\sigma_m^2}{2\left[(kZ_i^\tau)^{-1}+1\right]} \left(B_w - B_p\left(\frac{\alpha+1-\theta}{\alpha}\right) + \frac{1}{\alpha} - 1 \right) \right) \times \\
& \times \exp\left(-\left[\frac{\left(B_w - B_p\left(\frac{\alpha+1-\theta}{\alpha}\right) + \frac{1}{\alpha}\right)(kZ_i^\tau)^{-1}}{(kZ_i^\tau)^{-1}+1} - \frac{\theta}{\alpha} b_p \right] \frac{\sigma_m^2}{2} \right) \times \\
& \times \exp\left(\frac{\left[\frac{\left(B_w - B_p\left(\frac{\alpha+1-\theta}{\alpha}\right) + \frac{1}{\alpha}\right)\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1} - \frac{\theta}{\alpha} b_p \right]^2}{2} (kZ_i^\tau + 1)\, \sigma_m^2 \right)
\end{aligned}
$$

In order to solve for the optimal attention allocation we want to take the derivative of this with respect to the attention level Z. With this purpose we first take the derivatives of the elements defining the price a_p and b_p from (28) and (29),

$$
\begin{aligned}
\frac{\partial a_p}{\partial Z_i} =& \left[\frac{\alpha k^{-1}\tau}{(\theta-1)(1-\alpha)+1} \frac{\left\{\left[\frac{1}{\alpha} + B_w + B_{p1}\right]\left[\frac{1}{\alpha} + B_w + B_{p1} - 1\right] - \left[1 + B_{p2}\right] B_{p2}\right\} \frac{\sigma_m^2}{2}}{\left[(kZ_i^\tau)^{-1}+1\right]^2} Z_i^{-\tau-1} \right] + \\
& - \frac{1}{(\theta-1)(1-\alpha)+1} \frac{\eta}{(1+\eta Z_i)} \\
\frac{\partial b_p}{\partial Z_i} =& \frac{\alpha}{(\theta-1)(1-\alpha)+1} \left[\frac{1}{\alpha} - 1 + B_w + B_p \left(\frac{(\theta-1)(1-\alpha)}{\alpha} \right) \right] \frac{k\tau Z_i^{\tau-1}}{\left[1+kZ_i^\tau\right]^2}.
\end{aligned}
$$

Finally, taking the derivative of profits with respect to attention, we obtain the first order condition

$$
\begin{aligned}
\frac{\partial E\Pi}{\partial Z_i} =& \frac{\bar{M}}{\gamma_m}\exp\left(\psi_1+\psi_2+\psi_3\right)\left[\left(1-\theta\right)\frac{\partial a_p}{\partial Z_i} - k\tau\frac{\left(2-\theta\right)B_p\left[1-\left(2-\theta\right)B_p\right]\sigma_m^2}{2\left[\left(kZ_i^\tau\right)^{-1}+1\right]^2\left(kZ_i^\tau\right)^2}Z_i^{\tau-1}\right]+ \\
&+\frac{\bar{M}}{\gamma_m}\exp\left(\psi_1+\psi_2+\psi_3\right)\left\{\frac{\left[1-\left(2-\theta\right)B_p\right]}{\left[1+kZ_i^\tau\right]^2}\kappa\tau Z_i^{\tau-1}\frac{\sigma_m^2}{2}-\left(1-\theta\right)\frac{\partial b_p}{\partial Z_i}\frac{\sigma_m^2}{2}\right\} \\
&+\frac{\bar{M}}{\gamma_m}\exp\left(\psi_1+\psi_2+\psi_3\right)\left(\frac{\left[\frac{\left[1-(2-\theta)B_p\right]\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1}+\left(1-\theta\right)b_p\right]^2}{2}\left(k\tau Z_i^{\tau-1}\right)\sigma_m^2\right) \\
&+\frac{\bar{M}}{\gamma_m}\exp\left(\psi_1+\psi_2+\psi_3\right)\left[-\frac{\left[1-\left(2-\theta\right)B_p\right]\left(kZ_i^\tau\right)^{-1}}{\left(kZ_i^\tau\right)^{-1}+1}+\left(1-\theta\right)b_p\right]\times \\
&\times\left(\left[\frac{\left[1-\left(2-\theta\right)B_p\right]\left(k\tau Z_i^{\tau-1}\right)}{\left(1+kZ_i^\tau\right)^2}+\left(1-\theta\right)\frac{\partial b_p}{\partial Z_i}\right]\left(1+kZ_i^\tau\right)\sigma_m^2\right) \\
&+\frac{1}{\alpha}\eta\left(\frac{M}{\bar{A}\left(1+\eta Z_i\right)\gamma_m}\right)^{\frac{1}{\alpha}}\exp\left(\psi 4+\psi_5+\psi_6+\psi_7\right)\left(1+\eta Z_i\right)^{-1}+ \\
&-\left(\frac{M}{\bar{A}\left(1+\eta Z_i\right)\gamma_m}\right)^{\frac{1}{\alpha}}\exp\left(\psi 4+\psi_5+\psi_6+\psi_7\right)\left[-\frac{\theta}{\alpha}\frac{\partial a_p}{\partial Z_i}\right]+ \\
&-\left(\frac{M}{\bar{A}\left(1+\eta Z_i\right)\gamma_m}\right)^{\frac{1}{\alpha}}\exp\left(\psi 4+\psi_5+\psi_6+\psi_7\right)\left[\frac{\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}\right)\sigma_m^2}{2\left[\left(kZ_i^\tau\right)^{-1}+1\right]^2\left(kZ_i^\tau\right)^2}k\tau Z_i^{\tau-1}\right]\times \\
&\times\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}-1\right) \\
&-\left(\frac{M}{\bar{A}\left(1+\eta Z_i\right)\gamma_m}\right)^{\frac{1}{\alpha}}\exp\left(\psi 4+\psi_5+\psi_6+\psi_7\right)\times \\
&\left[\frac{\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}\right)\left(k\tau Z_i^{\tau-1}\right)}{\left(1+kZ_i^\tau\right)^2}+\frac{\theta}{\alpha}\frac{\partial b_p}{\partial Z_i}\right]\frac{\sigma_m^2}{2} \\
&-\left(\frac{M}{\bar{A}\left(1+\eta Z_i\right)\gamma_m}\right)^{\frac{1}{\alpha}}\exp\left(\psi 4+\psi_5+\psi_6+\psi_7\right)\left[\frac{\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}\right)}{1+kZ_i^\tau}-\frac{\theta}{\alpha}b_p\right]\times \\
&\times\left[-\frac{\left(B_w-B_p\left(\frac{\alpha+1-\theta}{\alpha}\right)+\frac{1}{\alpha}\right)}{\left[1+kZ_i^\tau\right]^2}k\tau Z_i^{\tau-1}-\frac{\theta}{\alpha}\frac{\partial b_p}{\partial Z_i}\right]\left(1+kZ_i^\tau\right)\sigma_m^2
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\frac{M}{\bar{A}\left(1+\eta Z_i\right)\gamma_m} \right)^{\frac{1}{\alpha}} \exp\left(\psi 4 + \psi_5 + \psi_6 + \psi_7\right) \times \\
&\times \left[\frac{\left(B_w - B_p\left(\frac{\alpha+1-\theta}{\alpha}\right) + \frac{1}{\alpha}\right)}{1+kZ_i^{\tau}} - \frac{\theta}{\alpha} b_p \right]^2 \frac{k\tau Z_i^{\tau-1}\sigma_m^2}{2}. \qquad (30)
\end{aligned}
$$

We can now use this to compute (numerically) the optimal attention allocation.

Appendix C: Computation methodology

Given a level of monetary uncertainty (and fixing a seed),

1. Guess the wage function (25), i.e. guess elements A_w^0 and B_w^0
2. Solve for the optimal attention allocation Z using the first order condition (30), taking into that $Z \in [0,1]$. Also obtain the policy function for pricing given the signal, as defined by (28) and (29).
3. Generate large T number of monetary shocks . For each shock guess a wage level (possibly the one defined by the guess)
   (a) Using the policy function, simulate many firms. Obtain the equilibrium output, prices and labour demand and supply.
   (b) If the labour market clears, stop. Otherwise, try a new wage and restart (bisection method). Save the equilibrium wage.
4. Use the new equilibrium wage levels to do a regression of the form of (25), to obtain new elements A_w^1 and B_w^1. If these are significantly different from the guess A_w^0 and B_w^0 update the guess and go to step 1. Otherwise, stop and check that the fit is good enough (for example check that the R^2 of the regression is very close to one).

Appendix D: Variables' definitions and sources

Table 7: Variables

Name	Definition	Source
salf	Expected sales growth	WBES
serv	Service sector	WBES
frk_yn	Foreign owned	WBES
gvt_yn	Government owned	WBES
eco_pred	Economic Unpredictability	WBES
law_pred	Policy Unpredictability	WBES
ma5_rgdp	Real GDP per capita (5-year-average)	WDI
rgdpgrowth	Real GDP per capita growth	WDI
rgdpinitial	Initial GDP per capita	WDI
sdevinfl	Standard Deviation of Inflation (5-year)	WDI
sdevxrat	Standard Deviation of Exchange Rate (5-year)	WDI
sdevm2	Standard Deviation of M2 Growth (5-year)	WDI
pop_growth	Popularion Growth	WDI
govexp	Government Expenditure	WDI
inv	Investment	WDI
infl	Inflation	WDI
trade	Trade	WDI
legor_uk	UK legal origin	S
legor_fr	French legal origin	S
legor_ge	German legal origin	S
legor_sc	Scandinavian legal origin	S
legor_so	Socialist legal origin	S
pop	Population	BL
yr_sch	Years of Schooling	BL

WBES: World Business Environment Survey. **WDI:** World Development Indicators. **S:** Schleifer (*JEL*, 2008). **BL:** Barro-Lee.