Macroeconomic Theory II, Spring 2014: Homework 6

Problem 1

Consider Hansen-Rogerson's indivisible labor model we studied in class (where agents choose lotteries over employment/nonemployment and where there are competitive insurance markets against nonemployment) and, add on top of it, a dynamic neoclassical growth model with aggregate production function $F(K_t, N_t)$. K_t is the stock of capital and N_t the aggregate of efficiency-weighted hours worked. Assume households have intra-period utility

$$u(c_{it}, h_{it}) = \log(c_{it}) - \phi_i v(h_{it})$$

with $h_{it} \in \{0, \bar{h}\}$. Can you obtain a representative agent aggregation of the economy in the following three cases?

1) Agents differ in their initial wealth, but all the agents are equally productive (with productivity ε), and $\phi_i = \phi$, i.e., disutility of work is common across agents.

2) Agents differ in their disutility of work ϕ_i, but all have the same initial wealth, and the same labor productivity ε.

3) Agents differ in their labor productivity ε_i (fixed over time), but all agents have the same initial wealth, and the same disutility of work ϕ.

Problem 2

Consider an economy where households have period utility $u(c, 1 - h)$ where c is consumption and h is hours worked, and discount factor β. The economy is populated by $J + 1$ overlapping generations indexed by $j = 0, 1, .., J$. The size of the population of age $j > 1$ is φ^j and the total size of the population at each date t is normalized to 1.

Until age J^{ret} households work. Workers' efficiency units of labor evolve through the Markov chain $\pi(\varepsilon', \varepsilon)$. After age J^{ret}, households are retired since their efficiency units of labor are zero. The wealth of the deceased is confiscated by the government which redistributes it entirely to the newborn as a lump-sum transfer ϕ. The newborn draw their initial productivity level from the invariant distribution π^*.

Households save through a risk-free asset with rate of return r. Working households are free to borrow up to a limit $\bar{a}$, while retirees cannot borrow. Wages per efficiency unit

are w and are determined in the competitive labor market. Firms operate a standard CRS production function in capital K and aggregate efficiency-weighted hours N.

The government implements a pay-as-you-go social security system that pays to each retiree, every period, benefits b equal to a fraction ρ of her average earnings computed over her lifetime spell as a worker. The system is financed through a labor income tax τ paid by workers only. The government budget is balanced every period.

1) State the household problem in steady state in DP formulation. Define a stationary recursive competitive equilibrium for this economy.

As the economy is peacefully resting in its steady-state, the government announces that will abolish social-security benefits (i.e., set $b = 0$), but only for the new cohorts entering in the economy from that date onward, i.e., the pension system keeps surviving (and $b > 0$) only for existing cohorts until all their members are deceased.

2) Describe the household problem in the new steady-state with privatized social security.

3) Explain how you would compute the equilibrium of this economy during the transition induced by this announcement.

4) What would you expect to happen to equilibrium prices and allocations during the transition?